

In the final structure about half the positive charge is located at the iron atom and the other half distributed uniformly over both rings (see table). For comparison the charge at the iron atom in ferrocene is calculated to be $+0.3$. This result compares favourably with *Mössbauer* experiments [13]. Another interesting result of these calculations is that a planar structure (III) should have a triplet groundstate (1.4 eV difference between highest occupied and lowest unoccupied MO) [14] while the bent structure (V) should have a singlet ground state. Some experiments carried out by *Cais et al.* [15] may be explained in this way.

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BIBLIOGRAPHY

- [1] Reviews: (a) *M. Rausch*, Adv. Chemistry Ser. 37, 56 (1963); (b) *M. Rosenblum*, 'Chemistry of the Iron Group Metallocenes', part 1, John Wiley & Sons, New York 1965; (c) *M. Cais*, Record chem. Progr. 27, 177 (1966); (d) Organometal Chemistry Rev. 7, 435 (1966); (e) Third Jerusalem Symposium on Aromaticity, Pseudoaromaticity, Antiaromaticity 1970.
- [2] *M. Cais*, *J. J. Dannenberg*, *A. Eisenstadt*, *M. L. Levenberg* & *J. H. Richards*, Tetrahedron Lett. 1966, 1695; *J. Feinberg* & *M. Rosenblum*, J. Amer. chem. Soc. 91, 4342 (1969).
- [3] *E. A. Hill* & *J. H. Richards*, J. Amer. chem. Soc. 83, 3840 (1961).
- [4] *T. G. Traylor* & *J. C. Ware*, J. Amer. chem. Soc. 89, 2304 (1967).
- [5] *E. Heilbronner*, private communication.
- [6] *R. Hoffmann*, J. chem. Physics 39, 1397 (1963); 40, 2047, 2474, 2745 (1964).
- [7] *M. Zerner* & *M. Gouterman*, Theor. chim. Acta 4, 44 (1966).
- [8] *J. Hinze* & *H. H. Jaffé*, J. chem. Physics 84, 540 (1961).
- [9] *R. S. Mulliken*, J. Chim. physique 46, 497, 675 (1949); *M. Wolfsberg* & *L. Helmholz*, J. chem. Physics 20, 837 (1952).
- [10] *K. Ruedenberg*, *C. C. J. Roothaan* & *W. Jaunzemis*, J. chem. Physics 24, 201 (1956).
- [11] *J. W. Richardson*, *R. R. Powell* & *W. C. Nieuwpoort*, J. chem. Physics 38, 769 (1963).
- [12] *J. W. Richardson*, *W. C. Nieuwpoort*, *R. R. Powell* & *W. F. Edgell*, J. chem. Physics 36, 1057 (1962).
- [13] *J. J. Dannenberg* & *J. H. Richards*, Tetrahedron Lett. 47, 4747 (1967).
- [14] *R. Hoffmann*, *G. D. Zeiss* & *G. W. Van Dine*, J. Amer. chem. Soc. 90, 1485 (1968).
- [15] *M. Cais*, *A. Modiano* & *A. Raveh*, J. Amer. chem. Soc. 87, 5607 (1965).

125. A Semi-Empirical Model of the Energy Barrier of Proton Transfer Reactions¹⁾

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(27. 111. 71)

Summary. The energy barrier in proton transfer reactions is described by a *Johnston*-type equation (1) (n = order of bond to be broken). The barrier model is discussed in terms of free energies. The V_i values are free energies of ionic cleavage in aqueous solution of the X–H and Y–H bonds; they are computed from eqns. (4c) and (4d). The values of p_1 and p_2 affect curvature

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(absence or presence of maximum) and symmetry of the barrier. It is postulated that p_i is a typical constant of the reacting bond and can be transferred from one transition state to another.

With the aid of eqn. (1) and its first derivative, values of p_i and n_m (bond order at maximum of barrier) can be based on quantities determined experimentally, ΔG^\ddagger and ΔG . For O–H bonds, $p_i \approx 1.0$. For C–H bonds, p_i is larger than 1.0 and depends on the structure of the carbanionic moiety (influence of resonance and inductive effects).

As there cannot be a maximum if $p_1 = p_2 = 1.0$, the suggested model of the barrier leads to a better understanding why proton transfer must be 'fast' in some reactions and 'slow' in others. The computed values of n_m may be utilized to gain some insight into the nature of the transition states; they supply a basis for the discussion of primary hydrogen isotope effects.

Introduction. – In previous work on isotope effects in S_N2 reactions: $RX + Y^- \rightarrow RY + X^-$ [1], it was suggested that the energy barrier of a three center reaction can be approximately described by a *Johnston*-type equation (1) [2]:

$$V = V_1(1 - n^{p_1}) - V_2(1 - n)^{p_2}. \quad (1)$$

n is the RX bond order and $1-n$ is the RY bond order; V_1 and V_2 are the free energies of complete ionic cleavage, in aqueous solution, of the RX bond and the RY bond, respectively. It is assumed that V_i and p_i are characteristic constants of reacting bonds which are independent of the other nucleophile in the transition state. If a reasonable assumption can be made about the numerical values of V_i and p_i for one particular bond, it then is possible to compute the values of these constants for any other bond from experimental values of ΔG ($= V_1 - V_2$) and ΔG^\ddagger ($= V_{\max}$) for the reaction $RX + Y^- \rightleftharpoons RY + X^-$.

The discussion was carried out in terms of *Gibbs* free energies rather than potential energies because free energies can be obtained directly from experimental data of equilibrium and rate constants, and free energy changes may be considered as good approximations of potential energy changes [3] [4].

Though rate constants have been determined for a very large number of S_N2 reactions there was little previous interest in their equilibrium constants. It will be necessary to measure some equilibrium constants before the applicability of eqn. (1) to the barriers in S_N2 reactions can be rigorously ascertained.

Rate as well as equilibrium data are available for many proton transfer reactions. It suggests itself to apply the semi-empirical model to the energy barrier of the proton transfer process. Though the assumptions involved probably are not completely precise the semi-empirical model may lead to a better qualitative or semi-quantitative understanding of the factors which govern the height of the energy barrier and the approximate position of the proton on the reaction coordinate in the transition state of a proton transfer process.

In this paper, eqn. (1) is applied to the reaction $HX + Y \rightleftharpoons X + HY$ in aqueous solution. (Electric charges are not indicated in order to keep the formulation as general as possible.) V_1 and V_2 are the free energies (corrected for statistical factors) of complete ionic cleavage in aqueous solution of the H–X and H–Y bonds, to form $H^+(aq)$ and $X^-(aq)$ or $Y^-(aq)$, respectively, V_1 and V_2 include solvation energy changes.

The most important case is the one with $X = H_2O$. V_1 then refers to the process $H_3O^+(aq) \rightarrow H^+(aq) + H_2O(l)$. $H_3O^+(aq)$ represents the solvated hydroxonium ion in aqueous solution which is mainly $H_3O(OH_2)_3^+$ (plus secondary solvation) [5]. The

'free' proton in aqueous solution, $H^+(aq)$, is not covalently bonded to any particular H_2O molecule, but it is still under the influence of other solute-solvent interactions such as electrostatic forces caused by polarization of the solvent.

One might question the concept of the free proton in aqueous solution. However, the overall free energy of heterolytic cleavage, in aqueous solution, of the covalent bond between H^+ and H_2O is a quantity which is necessary for the treatment of the energy barrier according to eqn. (1) and it is defined by the assumptions involved in the semi-empirical model. The energy of removal of a proton from H_3O^+ must have a finite value and, consequently, the probability of existence of 'free' protons in aqueous solution must be larger than zero.

We still have to be sure that the 'free proton in solution' is sufficiently well defined: It may be expected that according to the laws of statistical thermodynamics there is some average distribution of various states with weak interactions of H^+ with the solvent. (States with covalent bonding to H_2O are excluded according to definition. Distances between free protons and the O atoms of water must be more than 10% larger than the equilibrium O–H bond length.) An attempt to evaluate the energy of removal of a proton from H_3O^+ in aqueous solution will be discussed below.

It is assumed that the sum of the bond orders of the two reacting bonds is constant and equal to unity. The significance of the exponents may be visualized as follows: If $p_1 = p_2 = 1$ the energy, V , must be linearly dependent on n , and it cannot have a maximum. (There would be no activation energy though the proton is strongly bonded.) If on the other hand $p_i > 1$, the energy set free in the fractional bond forming process must be less than the amount proportional to the bond order, since the reacting bond order is always smaller than 1 and, consequently, n^{p_i} must be smaller than n .

The conditions for a maximum are fulfilled if the first derivative of V is equal to zero and the second derivative is negative.

$$dV/dn = -p_1 V_1 n^{p_1-1} + p_2 V_2 (1-n)^{p_2-1} = 0, \quad (2)$$

$$d^2V/dn^2 = -p_1(p_1-1)V_1 n^{p_1-2} - p_2(p_2-1)V_2(1-n)^{p_2-2} < 0. \quad (3)$$

Provided the exponents are positive, the first derivative must pass through zero for a value of n between 0 and 1. A sufficient condition for a maximum is fulfilled if p_1 and p_2 are > 1 . In special cases, maxima may occur if only one of the p_i is greater than 1. A few examples of energy barriers have been calculated with $V_1 = V_2 = 70$ kcal: $V_{\max} \approx 5$ kcal for $p_1 = 1.0$ and $p_2 = 1.2$; $V_{\max} \approx 9$ kcal for $p_1 = p_2 = 1.2$, $V_{\max} \approx 20$ kcal for $p_1 = p_2 = 1.5$. In general, the higher the p_i the higher the energy barrier.

It is one of the main goals of this study to find relationships between p_i values and the chemical nature of the bonds to be broken or formed. If a hydrogen bond can be formed between donor and acceptor, the proton transfer reaction is fast and essentially diffusion-controlled because the energy barrier between the two equilibrium positions of the proton is low [6]. In such a case, p_1 and p_2 must lie between 1.0 and 1.1.

In the formation of a fractional bond between a proton and a carbanion, part of the energy gained must be expended for the desolvation of the combining ions. A similar desolvation process takes place in the fractional bond formation between a proton and an oxy-anion, but it is less important because the complete O–H bond is still polar and solvated by hydrogen bonding and dipole-dipole interactions with other solvent

molecules. Appreciable desolvation occurs already at low bond orders as the electric fields of the opposite charges begin to compensate each other at relatively long distances. On the other hand, covalent bonding requires shorter distances. Therefore, it is understandable that the energy set free can be 'less than proportional' to the bond order in the earlier phases of the protonation of a carbanion. Consequently, p_i may be greater than 1.0. However, it is difficult to estimate how important this is and if it can lead to significant differences between C protonation and O protonation. It must be expected that p_i depends on the solvent. This study refers to purely aqueous solutions only.

Values of p_i greater than 1.0 are also to be attributed to resonance effects. If a carbon base (carbanionic or neutral) contains a resonance system extending over several bonds, the unshared electron pair is not fully localized at the reacting carbon atom. If a proton is added to such an atom, some energy will be gained but delocalization energy must be expended before the bond order can approach unity. Consequently, as long as the reacting bond order (n_i) remains smaller than 1, the energy set free in the fractional bond forming process will be less than $n_i V_i$. This is well known and has been discussed previously, mainly with regard to the reverse reaction, e.g. proton removal from a C-H acid whose anion is resonance stabilized [7]. Experimental data are available which indicate that the kinetic acidity of such an acid is lower than expected. It does not correspond to the equilibrium acidity because stabilization of the carbanion moiety has not been appreciably developed in the transition state. It may be expected that p_i increases with increasing importance of resonance in the carbon base.

Evaluation of V_i and p_i from experimental data. - *Determination of V_i values.*

In the treatment of experimental data for a proton transfer reaction of XH with Y, it is necessary to consider statistical factors, q_{HX} and q_{HY} , the numbers of equivalent acidic protons in HX and HY, respectively. (Correspondingly, statistical factors for the numbers of equivalent basic positions in X and Y should be considered. They are equal to one, however, in almost all examples discussed in this study.) Eqn. (4) is valid for the free energy change of the proton transfer reaction:

$$\Delta G = V_1 - V_2 - RT \ln(q_{HX}/q_{HY}). \quad (4a)$$

Provided an experimental value of ΔG is available, this equation may be utilized for the calculation of V_2 if V_1 is known or conversely.

W , the free energy change of the reaction $H_3O^+(aq) \rightarrow H^+(aq) + H_2O(l)$, is closely related to the V_1 value of a proton transfer reaction of the type $H_3O^+ + Y^- \rightarrow H_2O + HY$;

$$W = V_1 - RT \ln 3. \quad (5)$$

If W is known V_2 can be computed from eqn. (4b):

$$V_2 = W - \Delta G + RT \ln q_{HY}. \quad (4b)$$

ΔG is obtained experimentally from the acidity constant of HY in water:

$$\Delta G = -RT \ln(1/K_{HY}) = -2.303 RT \text{p}K_{HY} \quad (6)$$

hence:

$$V_2 = W + 2.303 RT \text{p}K_{HY} + 2.303 RT \log q_{HY}. \quad (4c)$$

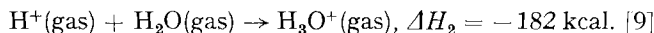
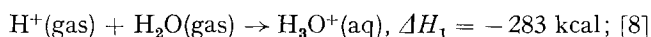
The analogous eqn. (4d) is correct for V_1 in the general case:

$$V_1 = W + 2.303 RT \text{p}K_{\text{HX}} + 2.303 RT \log q_{\text{HX}}. \quad (4d)$$

All calculations in this work are based on the following values:

$$W = 70.0 \text{ kcal}, V_1 = W + RT \ln 3 = 70.65 \text{ kcal}.$$

The value assumed for W is an estimate which has been derived from the reported enthalpy changes of the following reactions:



The transfer energy of the free proton from gas phase to aqueous solution has been estimated by a simple electrostatic calculation. Contributions of entropy changes to the value of W are considered to be relatively unimportant.

Evaluation of an estimate of W . Methods of determination of ΔH_1 and ΔH_2 from experimental data have been reviewed by Bell [10]. The energy of transfer of H_3O^+ from gas to liquid phase

$$\Delta H_3 = \Delta H_1 - \Delta H_2 = -101 \text{ kcal}.$$

The heat of transfer of one mole of water from gas to liquid phase $\Delta H_4 = -10.52 \text{ kcal}$ (see *e.g.* [11]). The enthalpy change, ΔH_6 , for the reaction $\text{H}^+(\text{aq}) + \text{H}_2\text{O}(\text{l}) \rightarrow \text{H}_3\text{O}^+(\text{aq})$ can be obtained by eqn. (7) provided ΔH_5 , the energy of transfer of 'free' H^+ from gas to liquid phase is known.

$$\Delta H_6 = \Delta H_2 + \Delta H_3 - \Delta H_4 - \Delta H_5 = \Delta H_1 - \Delta H_4 - \Delta H_5. \quad (7)$$

According to the laws of electrostatics, eqn. (8) is valid for the energy of transfer of a spherical ion, radius a , from vacuum (or gas at low pressure) to a medium with a dielectric constant > 1 :

$$\Delta E_{el} = -(N_0 e^2 / 2a) \cdot (1 - 1/D_e). \quad (8)$$

The effective dielectric constant, D_e , is $<$ the bulk dielectric constant, D , of the liquid because the concept of a completely homogeneous dielectric is not applicable to molecular dimensions. D_e approaches D for large values of a .

Eqn. (8) may be applied to the transfer of H_3O^+ from gas to liquid phase. The crystallographic radius of NH_4^+ is 1.48 \AA [12]. Considering the difference of the covalent radii of O and N [12], the radius of H_3O^+ is estimated as follows:

$$a_{(\text{H}_3\text{O}^+)} = 1.48 - 0.74 + 0.66 = 1.40 \text{ \AA}.$$

From $\Delta E_{el} = \Delta H_3 = -101 \text{ kcal}$, an effective dielectric constant $D_e = 7.00$ is computed. (In this treatment, the three H_2O molecules which form the primary solvation shell of H_3O^+ are considered as part of the dielectric.)

The radius of the 'free hydrogen ion in aqueous solution' is set equal to the value by which the radius of NH_4^+ exceeds the NH bond length: $a_{(\text{NH}_4^+)} - l_{\text{NH}} = 1.48 - 1.02 = 0.46 \text{ \AA}$. (It would lead to covalent interaction if the proton approaches the surface of a water molecule more closely.)

The effective dielectric constant is dependent on a . A simple exponential relationship is assumed for purposes of extrapolation:

$$D_e = D_\infty (1 - 10^{-\alpha a}). \quad (9)$$

$D_\infty = 78.3$ for water at 25° , $D_e = 7.0$ for H_3O^+ , $\alpha = 0.0290 \text{ \AA}^{-1}$. $D_e = 2.36$ then is computed from $a = 0.46 \text{ \AA}$ for the hydrogen ion. Eqn. (8) gives the electrostatic energy of transfer of 'free' H^+ from gas to liquid phase: $\Delta E_{el(\text{H}^+)} = -207 \text{ kcal}$. If it is assumed that $\Delta H_5 \approx \Delta E_{el(\text{H}^+)}$ we have all that is necessary for the computation of ΔH_6 from eqn. (7):

$$\Delta H_6 = -283 - (-10) - (-207) = -66 \text{ kcal}.$$

Since heterolytic cleavage of H_3O^+ does not involve separation or combination of opposite electric charges the entropy change must be relatively small and, therefore, $T\Delta S$ may be neglected in comparison to ΔH . (Similarly, it has been assumed that the entropies of transfer from gas to aqueous solution of H^+ and H_3O^+ partially cancel each other and the difference may be neglected in the overall computation.) Consequently: $\Delta H_6 \approx \Delta G_6 = -W$; the result has been rounded up to $W = 70 \text{ kcal}$. It is fully realized that this value is based on nothing but a rough estimate, though the order of magnitude may be correct.

Determination of p_i values. The height of the energy barrier in the symmetric transition state $[\text{H}_2\text{O} \cdots \text{H}^+ \cdots \text{OH}_2]^+$ is determined by the value of p_i ($= p_1 = p_2$) which refers to heterolytic cleavage of the $\text{H}_2\text{O} \cdots \text{H}^+$ bond. It can be expected that the exchange reaction $\text{H}_2\text{O}-\text{H}^+ + \text{OH}_2 \rightleftharpoons \text{H}_2\text{O} + \text{H}-\text{OH}_2^+$ is very fast and nearly diffusion-controlled. It is assumed that the barrier height is zero and, consequently: $p_i = 1.00$.

The following data are available for the calculation of energy barriers of reactions of the type $\text{H}_3\text{O}^+ + \text{Y} \rightleftharpoons \text{H}_2\text{O} + \text{HY}$ (electric charges on Y and HY not indicated): $V_1 = 70.65 \text{ kcal}$, $p_1 = 1.00$, and V_2 as calculated from pK_{HY} by eqn. (4c). V_{max} can be computed from eqn. (10) if the rate constant, k_f , of the reaction in forward direction has been measured.

$$V_{\text{max}} = \Delta G_f^\ddagger + RT \ln q_{\text{HX}} = -RT \ln(k_f/q_{\text{HX}}) + RT \ln(RT/N_0h). \quad (10)$$

(The statistical factor, $q_{\text{HX}} = 3$ if H_3O^+ is a proton donor.) Values of p_2 and n_m ($= n_{\text{max}}$) may now be calculated from eqns. (1) and (2):

$$V_{\text{max}} = V_1 [(1 - n_m^{p_1}) - (p_1/p_2) (1 - n_m)n_m^{p_1-1}]. \quad (11)$$

Eqn. (11) does not contain V_2 , hence:

$$p_2 = [p_1 (1 - n_m)n_m^{p_1-1}] / [1 - n_m^{p_1} - (V_{\text{max}}/V_1)]. \quad (11b)$$

Instead of solving eqns. (1) and (2) by successive approximations by an iterative method, the following method is applied advantageously: Starting from the input values of V_1 , p_1 , and V_{max} of the particular problem, p_2 is calculated (eqn. (11b)) in an electronic digital computer for all possible values of n_m from 0.005 to 0.995, with intervals of 0.005. For each pair of n_m and p_2 values, V_2 is calculated by eqn. (1b):

$$V_2 = [V_1 (1 - n_m^{p_1}) - V_{\text{max}}] / (1 - n_m)^{p_2}. \quad (1b)$$

Table 1. *Coefficients of eqn. (1) for the energy barriers in proton transfer reactions to carbon bases*

No.	Reaction	V_2 kcal	p_2	n_m	exp. data
For 1–18: $V_1 = 70.65$ kcal, $p_1 = 1.000$					
1	$H_3O^+ + (^-CH_2)CN$	104.75	1.309	0.883	[13]
2	$H_3O^+ + (^-CH_2)COOH$	103.35	1.337	0.863	[13]
3	$H_3O^+ + (^-CH)(CN)_2$	85.67	1.397	0.735	[13]
4	$H_3O^+ + (^-CH)(COOC_2H_5)_2$	88.51	1.478	0.725	[13]
5	$H_3O^+ + (^-CH_2)COCH_3$	98.34	1.384	0.819	[13]
6	$H_3O^+ + (^-CH)(COOC_2H_5)COCH_3$	84.97	1.376	0.738	[13]
7	$H_3O^+ + (^-CH)(COCH_3)_2$	82.68	1.535	0.665	[13]
8	$H_3O^+ + (^-C)(CH_3)(COCH_3)_2$	85.00	1.588	0.667	[13]
9	$H_3O^+ + (^-CH)(COC_6H_5)(COCH_3)$	83.22	1.526	0.672	[13]
10	$H_3O^+ + (^-C)(CH_3)(COOC_2H_5)(COCH_3)$	87.05	1.704	0.651	[21]
11	$H_3O^+ + (^-CHCl)(COCH_3)$	92.94	1.467	0.755	[13]
12	$H_3O^+ + (^-CH)(COCF_3)(COCH_3)$	76.82	1.844	0.561	[13]
13	$H_3O^+ + (^-CH_2)NO_2$	84.58	1.959	0.589	[13]
14	$H_3O^+ + (^-CH)(CH_3)NO_2$	82.14	2.105	0.555	[13]
15	$H_3O^+ + (^-CH)NO_2(COCH_3)$	77.36	1.776	0.576	[13]
16	$H_3O^+ + (^-CH)(NO_2)COOC_2H_5$	78.35	1.794	0.579	[13]
17	$H_3O^+ + 2, 4, 8$ -trimethylazulene	71.07	1.797	0.524	[14]
18	$H_3O^+ + 1, 3, 5$ -tri-MeO-benzene	63.34	1.963	0.444	[15]
For 35, 36: $V_1 = 89.51$ kcal, $p_1 = 1.087$					
35	$H_2O + (^-C \equiv C - C_6H_5)$	97.30	1.168	0.676	[22]
36	$H_2O + (^-CH)(CN)CH = CH - CH_2CN$	99.42	1.302	0.656	[23]

Table 2. *Coefficients of eqn. (1) for energy barriers in reactions of CH acids with oxygen bases*

No.	Reaction	V_1 kcal	p_1	V_2 kcal	p_2	n_m	exp. data
5	$CH_3COCH_3 + OH_2$	98.34	1.384	70.65	1.000	0.181	[13]
19	$CH_3COCH_3 + ^-OOC \cdot CH_3$	98.34	1.384	76.49	1.062	0.249	[16]
20	$CH_3COCH_3 + ^-OH$	98.34	1.384	89.51	1.087	0.375	[16]
8	$CH_3COCH(CH_3)COCH_3 + OH_2$	85.00	1.588	70.65	1.000	0.333	[13]
21	$CH_3COCH(CH_3)COCH_3 + ^-OOC \cdot CH_3$	85.00	1.588	76.49	1.048	0.395	[17]
13	$CH_3NO_2 + OH_2$	84.58	1.959	70.65	1.000	0.411	[13]
22	$CH_3NO_2 + ^-OOC \cdot CH_2Cl$	84.58	1.959	73.90	0.759	0.362	[18]
23	$CH_3NO_2 + ^-OOC \cdot CH_3$	84.58	1.959	76.49	0.816	0.398	[18]
24	$CH_3NO_2 + ^-OH$	84.58	1.959	89.51	0.939	0.516	[19]
15	$CH_3COCH_2NO_2 + OH_2$	77.36	1.776	70.65	1.000	0.424	[13]
25	$CH_3COCH_2NO_2 + ^-OOC \cdot CHCl_2$	77.36	1.776	71.74	0.962	0.423	[20]
26	$CH_3COCH_2NO_2 + ^-OOC \cdot CH_2Cl$	77.36	1.776	73.84	0.963	0.440	[20]
27	$CH_3COCH_2NO_2 + ^-OOC \cdot C_6H_5$	77.36	1.776	75.81	0.956	0.454	[20]
28	$CH_3COCH_2NO_2 + ^-OOC \cdot CH_3$	77.36	1.776	76.49	0.965	0.462	[20]
10	$CH_3COCH(CH_3)COOC_2H_5 + OH_2$	87.05	1.704	70.65	1.000	0.349	[21]
29	$CH_3COCH(CH_3)COOC_2H_5 + ^-OOC \cdot CHCl_2$	87.05	1.704	72.02	0.893	0.324	[21]
30	$CH_3COCH(CH_3)COOC_2H_5 + ^-OOC \cdot CH_2Cl$	87.05	1.704	73.84	0.893	0.337	[21]
31	$CH_3COCH(CH_3)COOC_2H_5 + ^-OOC \cdot CH_2CH_2Cl$	87.05	1.704	75.44	0.896	0.349	[21]
32	$CH_3COCH(CH_3)COOC_2H_5 + ^-OOC \cdot CH_3$	87.05	1.704	76.49	0.911	0.362	[21]
33	$CH_3COCH(CH_3)COOC_2H_5 + ^-OOC \cdot C(CH_3)_3$	87.05	1.704	76.85	0.907	0.363	[21]
34	$CH_3COCH(CH_3)COOC_2H_5 + HPO_4^{2-}$	87.05	1.704	79.40	0.944	0.395	[21]

The computer printout supplies a table of sets of values of n_m , p_2 , and V_2 . The results of n_m and p_2 for the particular energy barrier will be found on the same line as the correct value of V_2 which applies to the reaction under consideration; linear extrapolations may be carried out whenever necessary. This method is simple and it probably requires less computer time than calculation with a successive approximations computer program.

Results. On the basis of experimentally determined equilibrium and rate constants, values of n_m and p_2 were evaluated for 18 proton transfer reactions involving H_3O^+ and a carbon base (tab.1). Rates of reaction of H_3O^+ with various carbanions were computed from equilibrium and rate constants for the reverse reaction [13]. Rates of protonation of a few aromatic compounds (to form sigma complexes) are available directly from published results [14] [15].

In the next step, calculations were carried out for reactions of the type $XH + B^- \rightarrow X^- + HB$, with $B = OH^-$ or $RCOO^-$, for relevant experimental data see ref. [16]-[26]; V_1 and V_2 were computed from pK_{HX} and pK_{HY} according to eqn. (4d) and (4c)). Input values of p_1 were taken from the results for p_2 obtained in calculations for proton transfer from H_3O^+ to the same carbanion X^- . (For example, a value of $p_2 = 1.384$ was obtained from the data for the reaction of H_3O^+ with $CH_3COCH_2^-$. In the second series, calculations were carried out for proton transfer reactions from acetone to various oxy-anions, with $p_1 = 1.384$.) For results of calculations of the second series, see tab. 2.

Energy barriers of reactions between H_2O and very strongly basic carbanions, with $p_1 = 1.087$ (=result for p_2 in calc. no. 20, tab.2) are exemplified by reactions

 Table 3. Brönsted β exponents and primary isotope effects

No.	Reaction	n_2 (= n_{OH})	β	k_H/k_D exp. (25°)	stretch contribution calc.
1	$NC-CH_3 + OH_2$	0.883			6.75
2	$HOOC-CH_3 + OH_2$	0.863			6.91
5	$CH_3COCH_3 + OH_2$	0.819	} 0.88 [24]	3.5 [18]	7.11
19	$CH_3COCH_3 + ^-OOC-CH_3$	0.751		7.0 [18]	6.95
20	$CH_3COCH_3 + ^-OH$	0.625		10.2 [27]	7.28
8	$CH_3COCH(CH_3)COCH_3 + OH_2$	0.667		3.5 [17]	7.34
21	$CH_3COCH(CH_3)COCH_3 + ^-OOC-CH_3$	0.605		5.8 [17]	7.33
29	$CH_3COCHMeCOOEt + ^-OOC-CHCl_2$	0.676		3.85 [21]	
30	$CH_3COCHMeCOOEt + ^-OOC-CH_2Cl$	0.663		5.18 [21]	
31	$CH_3COCHMeCOOEt + ^-OOC-CH_2CH_2Cl$	0.651		5.72 [21]	
32	$CH_3COCHMeCOOEt + ^-OOC-CH_3$	0.638		5.92 [21]	
33	$CH_3COCHMeCOOEt + ^-OOC-C(CH_3)_3$	0.637		6.45 [21]	
7	$CH_3COCH_2COCH_3 + OH_2$	0.665	0.48 [25]		7.34
13	$CH_3NO_2 + OH_2$	0.589		3.8 [18]	7.34
22	$CH_3NO_2 + ^-OOC-CH_2Cl$	0.638	} 0.67 [26]	4.3 [18]	
23	$CH_3NO_2 + ^-OOC-CH_3$	0.602		6.5 [18]	7.34
24	$CH_3NO_2 + ^-OH$	0.484		10.3 [19]	7.33
18	$H_3O^+ + 1, 3, 5\text{-tri-MeO-benzene}$	(n_{CH}) 0.556	(α) 0.52 [15]		

no. 35, 36 in tab.1. Input values of V_{\max} were computed from equilibrium and rate constants of the reverse reaction [22] [23].

All experimental rate and equilibrium data refer to a temperature of 25° or in its vicinity, with the exception of those for nitroacetone which refer to 0°.

In tab.3, calculated bond orders n_2 of the new bond in the transition state are compared with *Brönsted* β values [15] [24] [25]. ($n_2 = n_{\text{OH}}$, with the exception of no.18 in which $n_2 = n_{\text{CH}}$ and corresponds to the *Brönsted* α .) Tab.3 also contains experimental values of kinetic primary deuterium isotope effects [26] and stretching vibrational contributions to the deuterium isotope effect which have been calculated from the energy barrier model.

In the calculations of the stretching vibrational contributions to the isotope effect, it is assumed that the stretching force constant of a reacting bond is proportional to its bond order in the transition state. The force constant f_{12} of the interaction of the X–H and H–Y stretches depends on the curvature of the barrier. As suggested by *Johnston* [2], the stretching force constants of the reacting bonds are calculated by eqns. (12)–(17):

$$f_{\text{XH}} = f_{11} = n_m F_{\text{XH}} \quad (12)$$

$$f_{\text{YH}} = f_{22} = (1 - n_m) F_{\text{YH}} \quad (13)$$

(F_{XH} and F_{YH} are the stretching force constants of the XH and YH bonds in stable molecules)

$$f_{12} = [f_{11}C^2 + f_{22} + (1 + C^2)f^*]/2C \quad (14)$$

$$C = (1 - n_m)/n_m \quad (15)$$

$$f^* = - (d^2V/dn^2)/(ds/dn)^2 \quad (16)$$

$$(ds/dn)^2 = (0.6/2.303)^2 [1/n^2 + 1/(1 - n)^2]. \quad (17)$$

Finally, stretching vibrational frequencies of reactants and transition states (simple three center models) and kinetic isotope effects are calculated by the *Wolfsberg-Schachtschneider* program [27].

Discussion. – *Values of V_i and p_i .* A more sophisticated treatment of the energy barrier in proton transfer reactions has been given by *Marcus* [28]. Somewhat similar ideas are applied in this work, with the special feature that an analytical expression (eqn. (1)), is introduced for the barrier which makes it possible to carry out numerical computations.

In the evaluation of the parameters for the energy barriers, a more or less arbitrary choice is made at the beginning concerning the values of V_1 and p_1 , which refer to cleavage of the bond between H^+ and H_2O . All other values of V_i and p_i then can be computed from experimental data. Since the energy barrier in proton transfer from H_3O^+ to H_2O is very low, p_i cannot be higher than 1.1. On the other hand, a value below 1.0 is meaningless in this case. Consequently, a very reasonable choice is made in assuming that $p_i = 1.00$. However, the selection of $W = 70$ kcal is not so well founded. It will be shown below that the suggested model of the energy barrier leads to meaningful results for p_i and n_m in almost all cases. Therefore, it may be concluded that either the selected value of W is in the right order of magnitude indeed – or selection of a more accurate value of W is not essential for the success of the treatment of the

experimental data on the basis of this model. Partial cancellation of errors certainly is possible because any change of W will alter V_1 and V_2 by the same amount, and the position of the maximum may not be shifted appreciably.

A similar answer can be given to another criticism which refers to the neglect of proton tunneling in eqn. (10). Though contributions to the overall rate by wave-mechanical tunneling are to be expected on theoretical grounds [2] [30], it is still not possible to arrive at quantitative predictions or to carry out experimental determinations of the tunneling rate. If a considerable part of the overall reaction occurs *via* proton tunneling the real energy barrier must be higher than the value computed from the experimental rate constant by eqn. (10). However, partial cancellation of deviations from eqn. (1) (as applied to V_{\max}) can be expected in comparisons of different reactions. Even though there may be some doubt about the accurate theoretical meaning of the V_{\max} values (from eqn. (10)) the suggested general validity of eqn. (1) still may be correct. It may be considered as a free energy relationship among experimental rate and equilibrium data.

It is postulated that the p_i values are characteristic constants of the reacting bonds, which are transferrable from one transition state to another. A direct cross check of this assumption has been possible in one case only: the values of p_2 obtained for reactions of 5 CH acids with acetate ion are: 1.06, 1.05, 0.965, 0.82, and 0.91 (tab.2) (the exceptionally low value of 0.82 applies to nitromethane). In general, values of p_2 between 0.90 and 1.09 are obtained for all proton transfer reactions from a CH acid to an oxygen base, with the exception of those for two reactions of nitromethane which are lower. These findings are encouraging since proton transfer reactions between oxygen bases are fast and must correspond to p_i values in the vicinity of 1.0.

The lowest p_i value (1.17) related to a reacting C-H bond has been found in the proton abstraction from phenylacetylene, where a proton is removed from a bond to a carbon atom with sp orbitals.

In all other examples of tab.1, the valence electrons at the reacting carbon atom are either in an sp^3 configuration or in transition between sp^2 and sp^3 (the configuration in the protonated form is always sp^3). The data in tab.1 indicate unambiguously that the p_i value depends on the amount of resonance stabilization in the carbanion. It increases from 1.30, for the two deprotonation reactions of alkyl cyanides, to approximately 2, for the reactions of nitroalkanes. The p_i value for the ionization of acetone is 1.38, it is increased to 1.53-1.59 in the β -diketones. Some data indicate the influence of inductive effects: *e.g.* $p_i = 1.47$ for chloroacetone and $p_i = 1.84$ for trifluoroacetylacetone.

The p_i values do not merely reflect the influence of structure on rates and equilibria. It is instructive to consider the β -diketones; introduction of a methyl group in position 3 of acetylacetone increases the pK by 2 units and the deprotonation rate by ca. 10^{2-3} , but leaves p_i relatively unchanged.

One may imagine that p_i will be in the region of 1.1 - 1.2 for non-activated aliphatic C-H bonds. Thus it appears that the most important cause of the slowness of proton transfer to and from carbon must lie in the resonance system of the carbanion, as explained in the introduction. If there is no activating group, proton transfer to the

carbanion must be relatively fast (proton removal from a non-activated C–H bond is slow because of the very high value of V_1).

The n_m values. There is much general interest in the position of the maximum of the energy barrier as defined by the bond order n_m (referring to the bond to be broken). A few general conclusions may be drawn from eqn. (1):

In the case that $V_1 = V_2$, $n_m < 0.5$ if $p_1 < p_2$, and $n_m > 0.5$ if $p_1 > p_2$.

In the case that $p_1 = p_2 (> 1)$, $n_m < 0.5$ if $V_1 > V_2$, and $n_m > 0.5$ if $V_1 < V_2$.

The latter statement is identical with the *Hammond* postulate [31]. In many proton transfer reactions from an OH acid to a carbanion, the influence of p_2 on n_m is opposed to that of V_2 since $V_2 > V_1$ and $p_2 > p_1$. However, sample calculations indicate that the influence of p_i on the position of the maximum is smaller, therefore, agreement with the qualitative predictions of the *Hammond* postulate may be expected in most cases. On the other hand, if transition states with H_3O^+ and various carbanions are compared with each other, p_2 usually increases with decreasing V_2 : both changes affect n_m in the same direction.

Results of n_m computed for reactions of H_3O^+ with carbanions range from 0.55 to 0.88 (tab. 1). The n_m values in transition states which contain anions of nitroalkanes are lower than those referring to anions of carbonyl compounds with similar pK . This is due to the exceptionally high p_2 values for the ionization of C–H bonds in nitroalkanes.

The basicity of 2,4,8-trimethylazulene is comparable to, and the basicity of 1,3,5-trimethoxybenzene is lower than that of water, and the values of n_m are 0.52 and 0.44, respectively, as expected.

In the transition states of the reactions of CH acids with various oxygen bases, n_m refers to the C–H bond order and $n_2 (= 1 - n_m)$ to the O–H bond order. According to the values recorded in tab. 2, n_m increases with increasing basicity of the oxy-anion as expected. The result for n_m in reaction no. 24 confirms the conclusion recently drawn by *Bordwell et al.* [32] from other experimental evidence.

As can be seen from tab. 3, there is approximate agreement between values of n_2 and experimental *Brönsted* β coefficients in some cases. It should be emphasized, however, that the interpretation of the *Brönsted* β as the degree of proton transfer in the transition state is somewhat dubious, particularly as the degree of proton transfer may differ in reactions with bases of different strengths.

Primary hydrogen isotope effects. The relationship between primary hydrogen isotope effect and degree of proton transfer in the transition state (reacting bond orders) has been discussed by various authors [33] [34]. In all theoretical treatments, the stretching force constants f_{XH} and f_{YH} in the transition state $[X \cdots H^+ \cdots Y]^\ddagger$ are linearly dependent on the reacting bond orders. If $f_{XH} = f_{YH}$ (corresponding to $n_m \approx 0.5$) the stretching motions of the hydrogen contribute very little to the vibrational zero point energy of the transition state, and consequently, k_H/k_D must be at its maximum value. On the other hand, the stretching vibrational contribution to the primary isotope effect must be lower for reactant-like ($f_{XH} > f_{YH}$) or product-like ($f_{XH} < f_{YH}$) transition states.

Bell & Goodall [19] plotted $\log(k_H/k_D)$ versus the difference of pK values of donor and acceptor for 20 reactions between CH acids (aliphatic carbonyl or nitro compounds)

and oxygen bases. They found that the isotope effect passes through a maximum near $\Delta pK = 0$. A similar observation has been made by *Longridge & Long* [14] for the hydrogen isotope effect in proton transfer reactions from protonated aromatic compounds (σ complexes) to oxygen bases. Furthermore, *Kresge et al.* [35] observed a maximum in a plot of k_H/k_D versus $\log k_H$ for the acid catalyzed hydrolyses of various vinyl ethers. These findings are in agreement with theoretical predictions since it may be expected that the orders of both reacting bonds are approximately equal to one-half if the basicities of X and Y in the transition state are equal.

According to the results in tab. 3, calculated values of n_2 near 0.5 correspond to high experimental isotope effects and those much different from 0.5 correspond to low experimental isotope effects. However, the calculated stretching vibrational contributions to the isotope effect exhibit very little dependence on n_2 ; they are close to the maximum value even in examples with $n_2 = 0.82$ or 0.88. This is a consequence of the relatively high curvature of the barrier [34] as computed from the semi-empirical model. (The curvature of the barrier would be even higher if the tunneling contribution to the rate is factored out.)

This result is completely unexpected and somewhat disappointing as it contradicts the widely accepted explanation for the variation of the hydrogen isotope effect in different three center proton transfer reactions. It follows that the observed low isotope effects in some reactions cannot be caused by disparities of the transition state stretching force constants f_{XH} and f_{YH} . It appears that the dependence of k_H/k_D on n_m (or n_2 , respectively) is a consequence of relationships between reacting bond orders and transition state bending force constants. A similar suggestion has been made previously by *Bell* [21 b]. Presumably, the sum of the bending force constants is low for $n_m = 0.5$ and high for n_m much different from 0.5, in such a way that its contribution to the isotope effect is not compensated by that of the bending force constants of the reactant.

The importance of bending force constants for the value of the primary hydrogen isotope effect has been demonstrated recently by *Kresge & Chiang* [36]: In a proton transfer reaction from HF as a donor, k_H/k_D is relatively low because there are no bending vibrations in the diatomic reactant and, consequently, the isotopic zero point energy difference of the bending vibrations of the transition state cannot be compensated by a zero point energy difference of bending vibrations in the reactant.

On the other hand, low values of the stretching vibrational contribution to the isotope effect are obtained from the semi-empirical model if n_m is either very close to zero (below 0.1) or very close to 1 (above 0.9). A very small value of n_m (referring to the OH bond order) probably occurs in the transition state of proton transfer from H_3O^+ to an alkene because the basicity of alkenes is very low in comparison to water. (pK values of simple alkenes as bases are unknown, therefore it is not possible to calculate n_m .) Accordingly, observed kinetic isotope effects in the acid catalyzed hydration of alkenes are close to 1 [37]. However, if the double bond is in conjugation with an aromatic ring or another double bond, the basicity of the alkene may be much higher and, consequently, a reacting bond order closer to 0.5 and a higher kinetic isotope effect are expected for the proton transfer reaction. Indeed, a hydrogen isotope effect of $k_H/k_D = 3.0$ is found in the acid catalyzed hydration of 1-phenyl-1,3-butadiene [38].

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BIBLIOGRAPHY

- [1] *A. V. Willi*, *Z. physik. Chem.* [N.F.] **66**, 317 (1969).
- [2] *H. S. Johnston*, *Advances chem. Physics* **3**, 131 (1960).
- [3] *L. P. Hammett*, 'Physical Organic Chemistry', McGraw-Hill, New York 1940.
- [4] *M. G. Evans & M. Polanyi*, *Trans. Farad. Soc.* **32**, 1333 (1936).
- [5] *E. Wicke, M. Eigen & Th. Ackermann*, *Z. physik. Chem.* [N.F.] **7**, 340 (1954).
- [6] *M. Eigen*, *Angew. Chem.* **75**, 489 (1963); *Discussions Farad. Soc.* **39**, 7 (1965).
- [7] *R. P. Bell*, *J. physik. Coll. Chem.* **55**, 885 (1951).
- [8] *E. C. Baughan*, *J. chem. Soc.* **1940**, 1403.
- [9] *J. Sherman*, *Chem. Reviews* **11**, 164 (1932).
- [10] *R. P. Bell*, 'The Proton in Chemistry', p.18–24, Cornell University Press, Ithaca, New York 1959; further references therein.
- [11] *F. Daniels & R. A. Alberty*, 'Physical Chemistry', 3rd ed., p. 51, Wiley, New York 1966.
- [12] *L. Pauling*, 'The Nature of the Chemical Bond', 3rd ed., p. 224, 226, 518, Cornell University Press, Ithaca, New York 1960.
- [13] *Op. cit.* [10], p.161–162; *R. G. Pearson & R. L. Dillon*, *J. Amer. chem. Soc.* **75**, 2439 (1953), further references therein.
- [14] *J. L. Longridge & F. A. Long*, *J. Amer. chem. Soc.* **89**, 1292 (1967).
- [15] *A. J. Kresge & Y. Chiang*, *J. Amer. chem. Soc.* **89**, 4411 (1967).
- [16] *R. P. Bell, G. R. Hillier, J. W. Mansfield & D. G. Street*, *J. chem. Soc. (B)* **1967**, 827; *R. P. Bell & P. W. Smith*, *ibid.* **1966**, 241; *R. P. Bell & H. C. Longuet-Higgins*, *ibid.* **1946**, 636.
- [17] *F. A. Long & D. Watson*, *J. chem. Soc.* **1958**, 2019.
- [18] *O. Reitz*, *Z. physik. Chem.* **A176**, 363 (1936); *O. Reitz & J. Kopp*, *ibid.* **184**, 429 (1939).
- [19] *R. P. Bell & D. M. Goodall*, *Proc. Roy. Soc. A* **294**, 273 (1966).
- [20] *R. P. Bell & R. R. Robinson*, *Proc. Roy. Soc. A* **270**, 411 (1962).
- [21] a) *R. P. Bell & J. E. Crooks*, *Proc. Roy. Soc. A* **286**, 285 (1965). b) *R. P. Bell*, *Discussions Farad. Soc.* **39**, 16 (1965).
- [22] *E. A. Halevi & F. A. Long*, *J. Amer. chem. Soc.* **83**, 2809 (1961).
- [23] *E. A. Walters & F. A. Long*, *J. Amer. chem. Soc.* **91**, 3733 (1969).
- [24] *R. P. Bell & O. M. Lidwell*, *Proc. Roy. Soc. A* **176**, 88 (1940); *op. cit.* [10], p.172.
- [25] *R. P. Bell, E. Gelles & E. Möller*, *Proc. Roy. Soc. A* **198**, 310 (1949); *op. cit.* [10], p. 172; [21] a), p.298.
- [26] *Op. cit.* [10], p.201.
- [27] *M. Wolfsberg & M. J. Stern*, *Pure Appl. Chemistry* **8**, 225 (1964).
- [28] *R. A. Marcus*, *J. physik. Chemistry* **72**, 891 (1968); *J. Amer. chem. Soc.* **91**, 7224 (1969).
- [29] *Op. cit.* [10], further references therein.
- [30] *E. F. Caldin & M. Casparian*, *Discussions Farad. Soc.* **39**, 25 (1965).
- [31] *G. S. Hammond*, *J. Amer. chem. Soc.* **77**, 334 (1955).
- [32] *F. G. Bordwell, W. J. Boyle & K. C. Yee*, *J. Amer. chem. Soc.* **92**, 5926 (1970).
- [33] *F. H. Westheimer*, *Chem. Reviews* **61**, 265 (1961); *L. Melander*, 'Isotope Effects on Reaction Rates', p.24–32, Ronald Press, New York 1960; *J. Bigeleisen*, *Pure Appl. Chemistry* **8**, 217 (1964).
- [34] *A. V. Willi & M. Wolfsberg*, *Chem. & Industry* **1964**, 2097.
- [35] *A. J. Kresge, D. S. Sagatys & H. L. Chen*, *J. Amer. chem. Soc.* **90**, 4174 (1968).
- [36] *A. J. Kresge & Y. Chiang*, *J. Amer. chem. Soc.* **91**, 1025 (1969).
- [37] *E. L. Pwlee & R. W. Taft*, *J. Amer. chem. Soc.* **78**, 5807 (1956).
- [38] *Y. Pocker & M. J. Hill*, *J. Amer. chem. Soc.* **91**, 7154 (1969).